Problem 1. (11 pt) Consider an AM transmitter whose transmitted signal is constructed from the message by

A

$$x_{\text{AM}}(t) = 5\cos(100\pi t) + m(t)\cos(100\pi t).$$

(a) (9 pt, ENRPr) We consider three cases with different modulation indexes. Their values are specified in the first column of the table below. Suppose the message is $m(t) = \alpha\cos(10\pi t)$.

$\alpha = m_p$

$P_m = \frac{\alpha^2}{2}$

(i) (3 pt) For each case, find the value of α which yields the specified modulation index. Put your answer in the second column of the table.

(ii) (3 pt) In the third column of the table, indicate (by writing a Y(es) or an N(o)) whether phase reversal occurs in each case.

(iii) (3 pt) In the fourth column, calculate the corresponding value of the power efficiency.

$\mu = \frac{m_p}{A}$

$m_p = \mu A$

μ

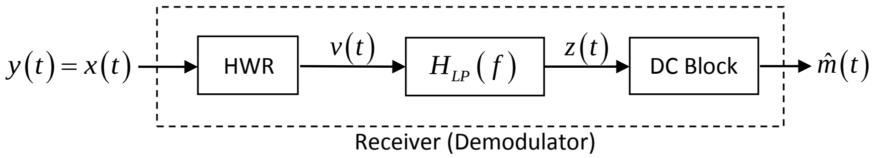
Mod. index	α	Phase Reversal	Power Eff.
75%	3.75	N	
100%	5	N	
125%	6.25	Y	

power eff. $= \dfrac{\frac{1}{2}P_m}{\frac{1}{2}A^2 + \frac{1}{2}P_m} = \dfrac{1}{1+\frac{A^2}{P_m}} = \dfrac{1}{1+\frac{A^2}{\frac{1}{2}\alpha^2}} = \dfrac{1}{1+\frac{2}{\mu^2}}$

(b) (1 pt) Suppose $m(t) = \cos(10\pi t) + 2\cos(30\pi t)$. What is the value of the modulation index?

(c) (1 pt) What is the best value of power efficiency that we can achieve without phase reversal?

Problem 2. (5 pt) Consider an AM receiver shown in the figure below:



Suppose the received signal $y(t)$ is the same as the transmitted signal which is given by

$$x(t) = (6 + 16m(t))\cos(2\pi f_c t) \quad \text{where} \quad f_c = 10^5 \text{ Hz}.$$

As usual, the message is band-limited to $B \ll f_c$. The half-wave rectifier input-output relation is described by a function $w(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$

The frequency response of the filter is $H_{LP}(f) = \begin{cases} g, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$

(a) (2 pt) The receiver above is a *rectifier detector*. To recover $m(t)$ back by this receiver, state the restriction that we need to impose on the message $m(t)$.

(b) (3* pt) Assume that appropriate restriction was made in part (a).

(i) (2 pt) Find $z(t)$. (Your answer will still depend on the constant g.)

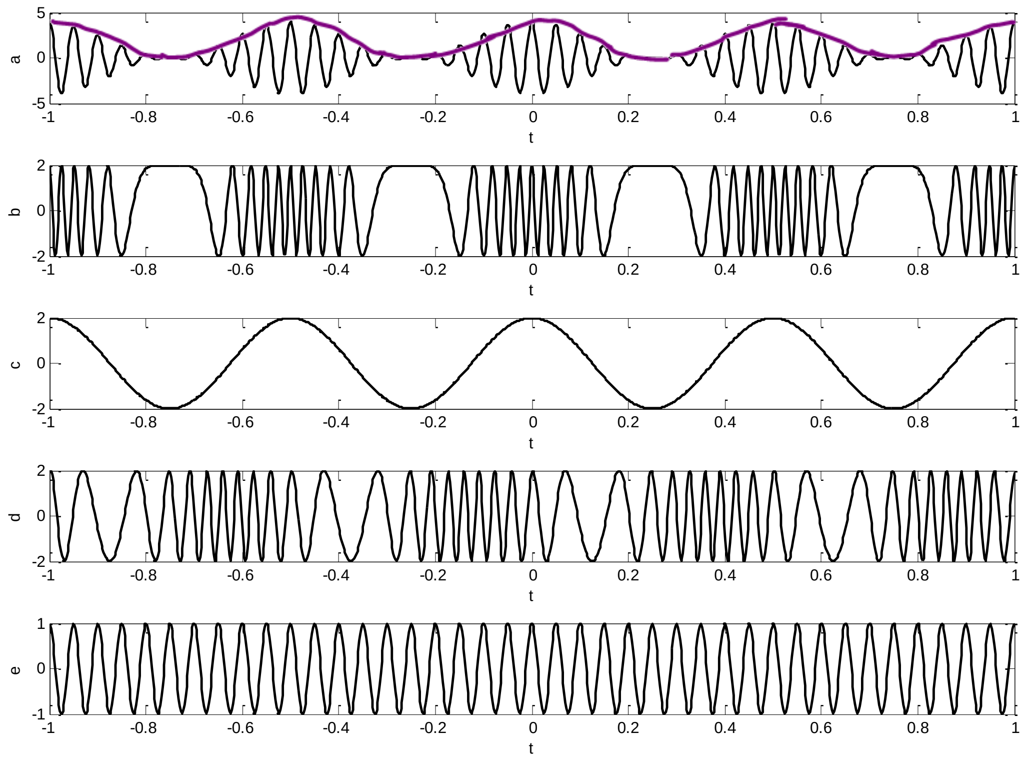
(ii) (1 pt) Find the constant g which makes $\hat{m}(t) = m(t)$.

Problem 4. (7 pt) Consider the five plots below. One of them is the baseband message signal $m(t)$. One of them is the sinusoid $\cos(2\pi f_c t)$ at the carrier frequency f_c. The message modulates the carrier signal $A\cos(2\pi f_c t)$, producing the other three plots which are the modulated signals $x_{\mathrm{AM}}(t)$, $x_{\mathrm{FM}}(t)$, and $x_{\mathrm{PM}}(t)$.

(a) (5 pt, ENRPa) In the boxes provided below, write down appropriate signal name ($m(t)$, $\cos(2\pi f_c t)$, $x_{\mathrm{AM}}(t)$, $x_{\mathrm{FM}}(t)$, and $x_{\mathrm{PM}}(t)$) to the left of its corresponding plot.

$(A + m(t))\cos(2\pi f_c t)$

$x_{AM}(t)$
$x_{FM}(t)$
$m(t)$
$x_{PM}(t)$
$\cos(2\pi f_c t)$



(b) (2 pt) What is the value of the modulation index μ used in $x_{\mathrm{AM}}(t)$?

100%

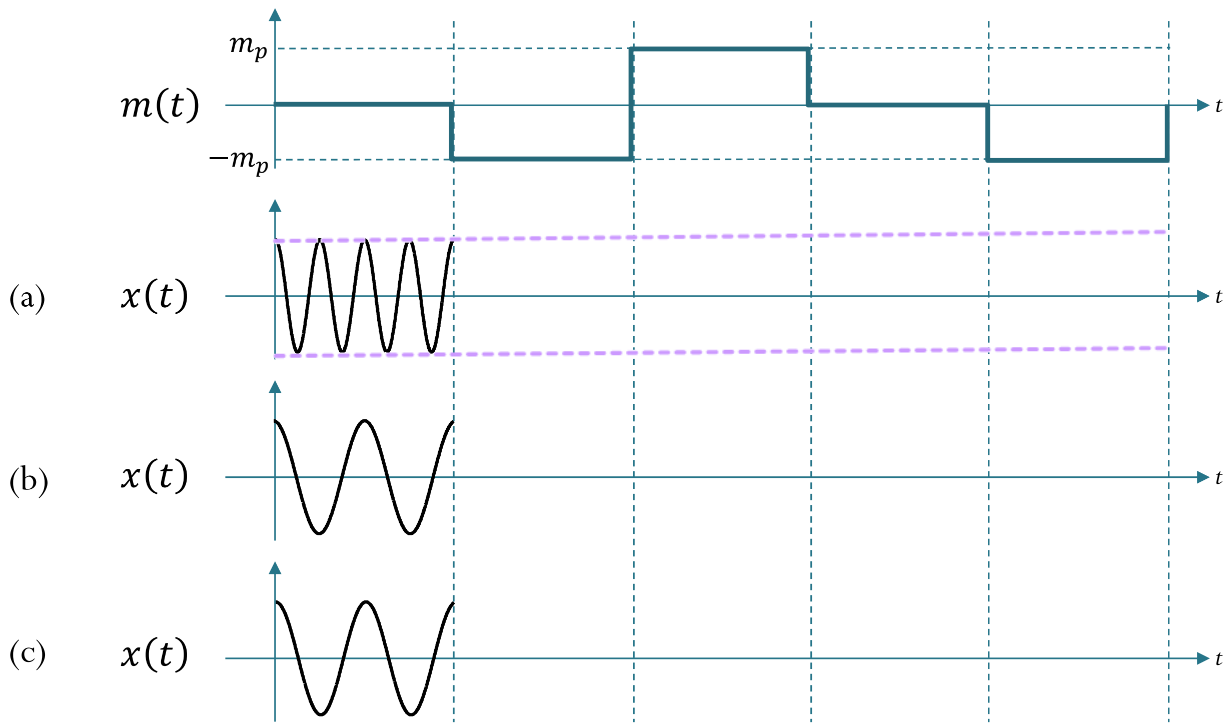
Problem 5. (13 pt, ENRPr) Consider the message $m(t)$ plotted below. For each part, ((a), (b), and (c)) below, your task is to carefully draw the remaining parts of the plots of the transmitted signals $x(t)$. The values of $x(t)$ during the first time interval were shown. Assume that

in part (a), the transmitted signal is generated using FM,

in part (b), the transmitted signal is generated using PM with $k_p = \frac{\pi}{m_p}$,

in part (c), the transmitted signal is generated using PM with $k_p = \frac{\pi}{2m_p}$.

For parts (b) and (c), you do not have to work on the last two intervals.



Make sure that the important "features" of the graphs are emphasized and labeled clearly.